

Practice Test E: Limits, Continuity, IVT

Name: _____

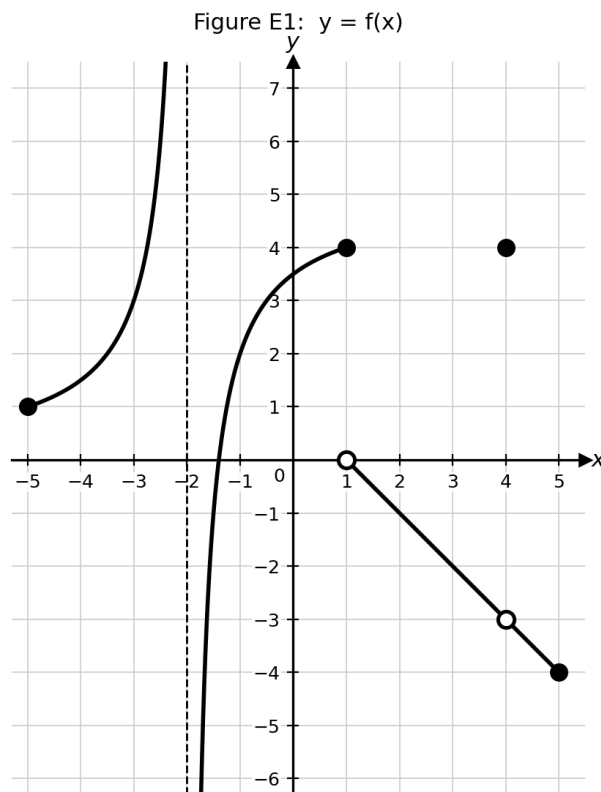
Time: 60 minutes No calculator 100 points

Answer exactly the question asked. Show work on every problem in Parts III–V.

If a limit is infinite, write ∞ or $-\infty$. If a limit does not exist for any other reason, write DNE.

If a function value does not exist, write *undefined*.

Part I. Graph. Questions 1(a)–1(g) refer to Figure E1.



1. Use the figure to find each value.

(a) $f(-2)$ _____

(b) $\lim_{x \rightarrow 1^-} f(x)$ _____

(c) $\lim_{x \rightarrow 1} f(x)$ _____

(d) $\lim_{x \rightarrow 4} f(x)$ _____

(e) $\lim_{x \rightarrow -2} f(x)$ _____

(f) $\lim_{x \rightarrow -2^+} f(x)$ _____

(g) $f(4)$ _____

(2 pts each, 14 pts)

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Part II. Table.

2. The table gives values of k . Assume the pattern continues as x approaches 2. **(2 each, 6 pts)**

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$k(x)$	7.51	7.9501	7.995001	1	8.005001	8.0501	8.51

(a) $\lim_{x \rightarrow 2^+} k(x)$ _____ (b) $\lim_{x \rightarrow 2} k(x)$ _____ (c) $k(2)$ _____

Part III. Algebra. Show work.

3. Evaluate $\lim_{x \rightarrow -3} \frac{x^2 - 5x + 4}{x^2 - 3x + 2}$, if it exists. **(5 pts)**

4. Evaluate $\lim_{x \rightarrow 4} \frac{\sqrt{x-3} - 2}{x-6}$, if it exists. **(5 pts)**

5. Evaluate $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x^2 - 2x - 3}$, if it exists. **(6 pts)**

6. Let $g(x) = \frac{x}{x^2 - 7x + 12}$. Evaluate each limit. **(3 each, 9 pts)**

(a) $\lim_{x \rightarrow 3^-} g(x)$ (b) $\lim_{x \rightarrow 3^+} g(x)$ (c) $\lim_{x \rightarrow 3} g(x)$

7. Let $p(x) = \begin{cases} 2 - x & x < 2 \\ -3 & x = 2 \\ 4 - x & x > 2 \end{cases}$ (2 each, 10 pts)

(a) $\lim_{x \rightarrow 6} p(x)$ (b) $\lim_{x \rightarrow 2^-} p(x)$ (c) $\lim_{x \rightarrow 2^+} p(x)$ (d) $\lim_{x \rightarrow 2} p(x)$ (e) $p(2)$

8. Let $r(x) = \frac{x^3 + 27}{x + 3}$. Find $r(-3)$, if it is defined. (3 pts)

Part IV. Continuity. For 9 and 10, check each of the three conditions in the definition of continuity at a point and state your conclusion.

9. Using Figure E1, determine whether f is continuous at $x = 4$. (6 pts)

10. Let $h(x) = \begin{cases} 3 - x & x < 3 \\ 1 & x = 3 \\ 6 - 2x & x > 3 \end{cases}$

Determine whether h is continuous at $x = 3$. (6 pts)

11. Using Figure E1, is f continuous on the closed interval $[-1, 1]$? Explain. (4 pts)

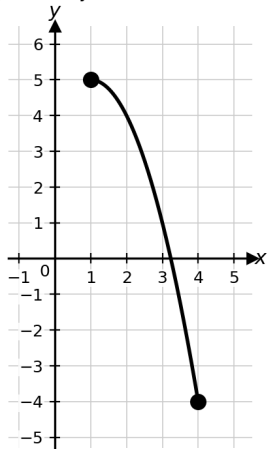
12. Let $q(x) = \frac{x-4}{x^2-2x-24}$. (3 each, 6 pts)
- (a) Is q continuous on $[-4, -2]$? Explain. (b) Is q continuous on $(6, 8]$? Explain.

Part V. Intermediate Value Theorem.

13. The function $h(x) = x^3 - 4x^2 - x - 1$ is continuous on $[-1, 2]$. Find $h(-1)$. (2 pts)
14. Let $f(x) = x^2 + x$ on the interval $[-4, -1]$. Does the Intermediate Value Theorem guarantee a number c in $(-4, -1)$ with $f(c) = 2$? If it does, find every such c in $(-4, -1)$. (6 pts)

15. Figure E2 shows the graph of r on its domain $[1, 4]$. Does the Intermediate Value Theorem guarantee a number c in $[1, 4]$ with $r(c) = 4$? If it does, find c . Explain. (4 pts)

Figure E2: $y = r(x)$, domain $[1, 4]$



16. True or False. Circle one. (4 each, 8 pts)
- (a) **T** **F** If f is continuous on $[1, 6]$, $f(1) = -2$, and $f(6) = 3$, then there is exactly one number c in $(1, 6)$ with $f(c) = 0$.
- (b) **T** **F** If g is continuous on $[2, 6]$, $g(2) = -1$, and $g(6) = 2$, then $g(c) = 0$ for at least one number c in $[2, 6]$.